

# THE SUBGROUPS OF $S_7$ .

<http://schmidt.nuigalway.ie/subgroups>

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### 1. THE CLASS OF SUBGROUPS OF ORDER 1 AND INDEX 5040.

**1. 1 [1], Normal, Orb 1<sup>7</sup>.**

**Contains 1 group, 1 class. DPY.**

**Below [2a](#)[21], [2b](#)[105], [2c](#)[105], [3a](#)[35], [3b](#)[140], [5](#)[126], [7](#)[120].**

## 2. THE 3 CLASSES OF SUBGROUPS OF ORDER 2 AND INDEX 2520.

2. **2a** [21], Nor  $S_5 \times 2$ , Orb  $2, 1^5$ . Contains 2 groups, 2 classes. PY.  
Above **1**. Below  $\underline{2^2c}$ [10],  $\underline{2^2e}$ [15],  $\underline{S_3a}$ [5],  $\underline{6b}$ [10],  $\underline{10}$ [6].
3. **2b** [105], Nor  $S_3 \times D_8$ , Orb  $2^2, 1^3$ . Contains 2 groups, 2 classes. DP.  
Above **1**. Below  $\underline{2^2a}$ ,  $\underline{2^2b}$ [3],  $\underline{2^2c}$ ,  $\underline{4a}$ ,  $\underline{4b}$ [3],  $\underline{2^2d}$ [3],  $\underline{2^2e}$ [3],  $\underline{6a}$ ,  $\underline{S_3d}$ [6],  $\underline{S_3e}$ [12],  $\underline{D_{10b}}$ [6].
4. **2c** [105], Nor  $S_4 \times 2b$ , Orb  $2^3, 1$ . Contains 2 groups, 2 classes. P.  
Above **1**. Below  $\underline{2^2d}$ [6],  $\underline{2^2e}$ [3],  $\underline{S_3b}$ [3],  $\underline{S_3c}$ [4],  $\underline{6c}$ [4],  $\underline{D_{10a}}$ [6],  $\underline{D_{14}}$ [8].

## 3. THE 2 CLASSES OF SUBGROUPS OF ORDER 3 AND INDEX 1680.

5. **3a** [35], Nor  $S_4 \times S_3$ , Orb  $3, 1^4$ . Contains 2 groups, 2 classes. D.  
Above **1**. Below  $\underline{S_3a}$ ,  $\underline{6a}$ [3],  $\underline{S_3b}$ [3],  $\underline{S_3d}$ [6],  $\underline{6b}$ [6],  $\underline{3^2}$ [4],  $\underline{A_4a}$ [4].
6. **3b** [140], Nor  $S_3^2a$ , Orb  $3^2, 1$ . Contains 2 groups, 2 classes. D.  
Above **1**. Below  $\underline{S_3c}$ ,  $\underline{6c}$ [3],  $\underline{S_3e}$ [3],  $\underline{3^2}$ ,  $\underline{A_4b}$ [2],  $\underline{A_4c}$ [3],  $\underline{7:3}$ [6].

## 4. THE 7 CLASSES OF SUBGROUPS OF ORDER 4 AND INDEX 1260.

7. **2^2a** [35], Nor  $S_4 \times S_3$ , Orb  $4, 1^3$ . Contains 5 groups, 3 classes. D.  
Above  $\underline{2b}$ [3]. Below  $\underline{2^3a}$ [3],  $\underline{D_{8a}}$ [3],  $\underline{D_{8d}}$ [9],  $\underline{A_4a}$ ,  $\underline{2^2 \times 3a}$ ,  $\underline{A_4b}$ [2].
8. **2^2b** [105], Nor  $S_4 \times 2b$ , Orb  $2^3, 1$ . Contains 5 groups, 3 classes. D.  
Above  $\underline{2b}$ [3]. Below  $\underline{2^3b}$ ,  $\underline{D_{8b}}$ [3],  $\underline{D_{8d}}$ [3],  $\underline{A_4c}$ ,  $\underline{D_{12a}}$ [3].
9. **2^2c** [105], Nor  $S_3 \times D_8$ , Orb  $2^2, 1^3$ . Contains 5 groups, 4 classes. PY.  
Above  $\underline{2b}$ ,  $\underline{2a}$ [2]. Below  $\underline{2^3b}$ [3],  $\underline{D_{8a}}$ ,  $\underline{D_{8c}}$ [3],  $\underline{2^2 \times 3b}$ ,  $\underline{D_{12d}}$ [6].
10. **4a** [105], Nor  $S_3 \times D_8$ , Orb  $4, 1^3$ . Contains 3 groups, 3 classes.  
Above  $\underline{2b}$ . Below  $\underline{D_{8a}}$ ,  $\underline{4 \times 2}$ [3],  $\underline{D_{8b}}$ [3],  $\underline{12}$ ,  $\underline{5:4b}$ [6].
11. **4b** [315], Nor  $D_8 \times 2$ , Orb  $4, 2, 1$ . Contains 3 groups, 3 classes.  
Above  $\underline{2b}$ . Below  $\underline{4 \times 2}$ ,  $\underline{D_{8c}}$ ,  $\underline{D_{8d}}$ ,  $\underline{2.S_3}$ ,  $\underline{5:4a}$ [2],  $\underline{3^2:4}$ [2].
12. **2^2d** [315], Nor  $D_8 \times 2$ , Orb  $4, 2, 1$ . Contains 5 groups, 4 classes.  
Above  $\underline{2c}$ [2],  $\underline{2b}$ . Below  $\underline{2^3a}$ ,  $\underline{D_{8b}}$ ,  $\underline{D_{8c}}$ ,  $\underline{D_{12b}}$ ,  $\underline{D_{12f}}$ [4].
13. **2^2e** [315], Nor  $D_8 \times 2$ , Orb  $2^3, 1$ . Contains 5 groups, 5 classes. P.  
Above  $\underline{2c}$ ,  $\underline{2b}$ ,  $\underline{2a}$ . Below  $\underline{2^3a}$ ,  $\underline{2^3b}$ ,  $\underline{4 \times 2}$ ,  $\underline{D_{12c}}$ ,  $\underline{D_{12e}}$ [2],  $\underline{D_{20}}$ [2].

## 5. THE CLASS OF SUBGROUPS OF ORDER 5 AND INDEX 1008.

14. **5** [126], Nor  $5:4 \times 2$ , Orb  $5, 1^2$ . Contains 2 groups, 2 classes. D.  
Above **1**. Below  $\underline{D_{10a}}$ ,  $\underline{D_{10b}}$ ,  $\underline{10}$ .

## 6. THE 8 CLASSES OF SUBGROUPS OF ORDER 6 AND INDEX 840.

15. **S<sub>3</sub>a** [35], Nor  $S_4 \times S_3$ , Der **3a**, Orb  $3, 1^4$ . Contains 6 groups, 4 classes. PY.  
Above  $\underline{3a}$ ,  $\underline{2a}$ [3]. Below  $\underline{D_{12c}}$ [3],  $\underline{D_{12d}}$ [6],  $\underline{S_3 \times 3b}$ [4],  $\underline{S_4a}$ [4].
16. **6a** [105], Nor  $S_3 \times D_8$ , Orb  $3, 2^2$ . Contains 4 groups, 4 classes. D.  
Above  $\underline{3a}$ ,  $\underline{2b}$ . Below  $\underline{2^2 \times 3a}$ ,  $\underline{2^2 \times 3b}$ ,  $\underline{D_{12a}}$ ,  $\underline{D_{12b}}$ ,  $\underline{2.S_3}$ ,  $\underline{D_{12c}}$ ,  $\underline{12}$ .
17. **S<sub>3</sub>b** [105], Nor  $S_3 \times D_8$ , Der **3a**, Orb  $3, 2^2$ . Contains 6 groups, 4 classes.  
Above  $\underline{3a}$ ,  $\underline{2c}$ [3]. Below  $\underline{D_{12b}}$ [2],  $\underline{D_{12c}}$ ,  $\underline{D_{12e}}$ [2].
18. **S<sub>3</sub>c** [140], Nor  $S_3^2a$ , Der **3b**, Orb  $6, 1$ . Contains 6 groups, 4 classes.  
Above  $\underline{3b}$ ,  $\underline{2c}$ [3]. Below  $\underline{D_{12f}}$ [3],  $\underline{S_3 \times 3a}$ ,  $\underline{S_4b}$ [3].
19. **S<sub>3</sub>d** [210], Nor  $S_3 \times 2^2b$ , Der **3a**, Orb  $3, 2, 1^2$ . Contains 6 groups, 4 classes.  
Above  $\underline{3a}$ ,  $\underline{2b}$ [3]. Below  $\underline{D_{12a}}$ ,  $\underline{D_{12d}}$ ,  $\underline{D_{12e}}$ ,  $\underline{3^2:2}$ [2],  $\underline{S_4c}$ [2],  $\underline{A_5a}$ .
20. **6b** [210], Nor  $S_3 \times 2^2b$ , Orb  $3, 2, 1^2$ . Contains 4 groups, 4 classes.  
Above  $\underline{3a}$ ,  $\underline{2a}$ . Below  $\underline{2^2 \times 3b}$ ,  $\underline{D_{12d}}$ ,  $\underline{D_{12e}}$ ,  $\underline{S_3 \times 3b}$ [2],  $\underline{A_4 \times 2a}$ [2].
21. **6c** [420], Nor  $D_{12f}$ , Orb  $6, 1$ . Contains 4 groups, 4 classes.  
Above  $\underline{3b}$ ,  $\underline{2c}$ . Below  $\underline{D_{12f}}$ ,  $\underline{S_3 \times 3a}$ ,  $\underline{A_4 \times 2b}$ ,  $\underline{7:6}$ [2].

22.  $S_3e$  [420], Nor  $D_{12}f$ , Der  $3b$ , Orb  $3^2, 1$ . Contains 6 groups, 4 classes. P.  
Above  $3b, 2b[3]$ . Below  $D_{12}f, 3^2:2, S_4d, S_4e[2], A_5b$ .

## 7. THE CLASS OF SUBGROUPS OF ORDER 7 AND INDEX 720.

23.  $7$  [120], Nor  $7:6$ , Orb  $7$ . Contains 2 groups, 2 classes. D.  
Above  $1$ . Below  $D_{14}, 7:3$ .

## 8. THE 7 CLASSES OF SUBGROUPS OF ORDER 8 AND INDEX 630.

24.  $2^3a$  [105], Nor  $S_4 \times 2a$ , Orb  $4, 2, 1$ . Contains 16 groups, 8 classes.  
Above  $2^2e[3], 2^2d[3], 2^2a$ . Below  $D_8 \times 2[3], S_3 \times 2^2a, A_4 \times 2a$ .

25.  $2^3b$  [105], Nor  $S_4 \times 2b$ , Orb  $2^3, 1$ . Contains 16 groups, 8 classes. PY.  
Above  $2^2e[3], 2^2c[3], 2^2b$ . Below  $D_8 \times 2[3], S_3 \times 2^2b[3], A_4 \times 2b$ .

26.  $D_8a$  [105], Nor  $S_3 \times D_8$ , Der  $2b$ , Orb  $4, 1^3$ . Contains 10 groups, 7 classes.  
Above  $4a, 2^2c, 2^2a$ . Below  $D_8 \times 2[3], S_4a, D_8 \times 3$ .

27.  $4 \times 2$  [315], Nor  $D_8 \times 2$ , Orb  $4, 2, 1$ . Contains 8 groups, 8 classes.  
Above  $2^2e, 4b, 4a$ . Below  $D_8 \times 2, S_3 \times 4, 5:4 \times 2[2]$ .

28.  $D_8b$  [315], Nor  $D_8 \times 2$ , Der  $2b$ , Orb  $4, 2, 1$ . Contains 10 groups, 7 classes.  
Above  $2^2d, 4a, 2^2b$ . Below  $D_8 \times 2, D_{24}, S_4b$ .

29.  $D_8c$  [315], Nor  $D_8 \times 2$ , Der  $2b$ , Orb  $4, 2, 1$ . Contains 10 groups, 8 classes. P.  
Above  $2^2d, 4b, 2^2c$ . Below  $D_8 \times 2, 2.D_{12}a, S_3:2[2]$ .

30.  $D_8d$  [315], Nor  $D_8 \times 2$ , Der  $2b$ , Orb  $4, 2, 1$ . Contains 10 groups, 6 classes.  
Above  $4b, 2^2b, 2^2a$ . Below  $D_8 \times 2, 2.D_{12}b, S_4c, S_4d, S_4e[2]$ .

## 9. THE CLASS OF SUBGROUPS OF ORDER 9 AND INDEX 560.

31.  $3^2$  [70], Nor  $S_3:2$ , Orb  $3^2, 1$ . Contains 6 groups, 4 classes. D.  
Above  $3b[2], 3a[2]$ . Below  $3^2:2, S_3 \times 3a[2], S_3 \times 3b[2], A_4 \times 3[2]$ .

## 10. THE 3 CLASSES OF SUBGROUPS OF ORDER 10 AND INDEX 504.

32.  $D_{10}a$  [126], Nor  $5:4 \times 2$ , Der  $5$ , Orb  $5, 2$ . Contains 8 groups, 4 classes.  
Above  $5, 2c[5]$ . Below  $D_{20}$ .

33.  $D_{10}b$  [126], Nor  $5:4 \times 2$ , Der  $5$ , Orb  $5, 1^2$ . Contains 8 groups, 4 classes.  
Above  $5, 2b[5]$ . Below  $5:4a, D_{20}, 5:4b, A_5a, A_5b[2]$ .

34.  $10$  [126], Nor  $5:4 \times 2$ , Orb  $5, 2$ . Contains 4 groups, 4 classes.  
Above  $5, 2a$ . Below  $D_{20}$ .

## 11. THE 13 CLASSES OF SUBGROUPS OF ORDER 12 AND INDEX 420.

35.  $A_4a$  [35], Nor  $S_4 \times S_3$ , Der  $2^2a$ , Orb  $4, 1^3$ . Contains 10 groups, 5 classes. D.  
Above  $2^2a, 3a[4]$ . Below  $S_4a, A_4 \times 2a[3], S_4c[3], A_4 \times 3, A_5a[3]$ .

36.  $2^2 \times 3a$  [35], Nor  $S_4 \times S_3$ , Orb  $4, 3$ . Contains 10 groups, 6 classes. D.  
Above  $6a[3], 2^2a$ . Below  $S_3 \times 2^2a, D_8 \times 3[3], 2.D_{12}b[3], A_4 \times 3$ .

37.  $A_4b$  [70], Nor  $(A_4 \times 3):2$ , Der  $2^2a$ , Orb  $4, 3$ . Contains 10 groups, 5 classes. D.  
Above  $2^2a, 3b[4]$ . Below  $S_4e[3], A_4 \times 3$ .

38.  $2^2 \times 3b$  [105], Nor  $S_3 \times D_8$ , Orb  $3, 2^2$ . Contains 10 groups, 8 classes.  
Above  $6b[2], 6a, 2^2c$ . Below  $S_3 \times 2^2b, D_8 \times 3, 2.D_{12}a$ .

39.  $A_4c$  [105], Nor  $S_4 \times 2b$ , Der  $2^2b$ , Orb  $6, 1$ . Contains 10 groups, 5 classes. D.  
Above  $2^2b, 3b[4]$ . Below  $S_4b, A_4 \times 2b, S_4d, A_5b[2]$ .

40.  $D_{12}a$  [105], Nor  $S_3 \times D_8$ , Der  $3a$ , Orb  $3, 2^2$ . Contains 16 groups, 7 classes.  
Above  $S_3d[2], 6a, 2^2b[3]$ . Below  $S_3 \times 2^2b, D_{24}, 2.D_{12}b, S_5b[2]$ .

41.  $D_{12}b$  [105], Nor  $S_3 \times D_8$ , Der  $3a$ , Orb  $4, 3$ . Contains 16 groups, 8 classes.  
Above  $S_3b[2], 6a, 2^2d[3]$ . Below  $S_3 \times 2^2a, D_{24}, 2.D_{12}a$ .

42.  $2.S_3$  [105], Nor  $S_3 \times D_8$ , Der 3a, Orb 4, 3. **Contains 8 groups, 6 classes.**  
Above [6a](#), [4b](#)[3]. Below [2.D<sub>12</sub>a](#), [S<sub>3</sub>×4](#), [2.D<sub>12</sub>b](#).
43.  $D_{12c}$  [105], Nor  $S_3 \times D_8$ , Der 3a, Orb 3, 2<sup>2</sup>. **Contains 16 groups, 10 classes. P.**  
Above [S<sub>3</sub>b](#), [6a](#), [S<sub>3</sub>a](#), [2<sup>2</sup>e](#)[3]. Below [S<sub>3</sub>×2<sup>2</sup>a](#), [S<sub>3</sub>×2<sup>2</sup>b](#), [S<sub>3</sub>×4](#).
44.  $12$  [105], Nor  $S_3 \times D_8$ , Orb 4, 3. **Contains 6 groups, 6 classes.**  
Above [6a](#), [4a](#). Below [D<sub>8</sub>×3](#), [D<sub>24</sub>](#), [S<sub>3</sub>×4](#).
45.  $D_{12d}$  [210], Nor  $S_3 \times 2^2b$ , Der 3a, Orb 3, 2, 1<sup>2</sup>. **Contains 16 groups, 9 classes. PY.**  
Above [6b](#), [S<sub>3</sub>d](#), [S<sub>3</sub>a](#), [2<sup>2</sup>c](#)[3]. Below [S<sub>3</sub>×2<sup>2</sup>b](#), [S<sub>3</sub><sup>2</sup>b](#)[2], [S<sub>4</sub>×2a](#)[2], [S<sub>5</sub>a](#).
46.  $D_{12e}$  [210], Nor  $S_3 \times 2^2b$ , Der 3a, Orb 3, 2<sup>2</sup>. **Contains 16 groups, 10 classes.**  
Above [6b](#), [S<sub>3</sub>d](#), [S<sub>3</sub>b](#), [2<sup>2</sup>e](#)[3]. Below [S<sub>3</sub>×2<sup>2</sup>b](#), [A<sub>5</sub>×2](#).
47.  $D_{12f}$  [420], Selfnor, Der 3b, Orb 6, 1. **Contains 16 groups, 9 classes. N.**  
Above [S<sub>3</sub>e](#), [6c](#), [S<sub>3</sub>c](#), [2<sup>2</sup>d](#)[3]. Below [S<sub>3</sub><sup>2</sup>a](#), [S<sub>4</sub>×2b](#), [S<sub>5</sub>c](#).

## 12. THE CLASS OF SUBGROUPS OF ORDER 14 AND INDEX 360.

48.  $D_{14}$  [120], Nor 7:6, Der 7, Orb 7. **Contains 10 groups, 4 classes.**  
Above [7](#), [2c](#)[7]. Below [7:6](#).

## 13. THE CLASS OF SUBGROUPS OF ORDER 16 AND INDEX 315.

49.  $D_8 \times 2$  [315], Selfnor, Der 2b, Orb 4, 2, 1. **Contains 35 groups, 19 classes. N.**  
Above [D<sub>8</sub>d](#), [D<sub>8</sub>c](#), [D<sub>8</sub>b](#), [4×2](#), [D<sub>8</sub>a](#), [2<sup>3</sup>b](#), [2<sup>3</sup>a](#). Below [S<sub>4</sub>×2a](#), [S<sub>3</sub>×D<sub>8</sub>](#), [S<sub>4</sub>×2b](#).

## 14. THE 3 CLASSES OF SUBGROUPS OF ORDER 18 AND INDEX 280.

50.  $3^2:2$  [70], Nor  $S_3:2$ , Der 3<sup>2</sup>, Orb 3<sup>2</sup>, 1. **Contains 28 groups, 8 classes. D.**  
Above [3<sup>2</sup>](#), [S<sub>3</sub>e](#)[6], [S<sub>3</sub>d](#)[6]. Below [S<sub>3</sub><sup>2</sup>a](#), [S<sub>3</sub><sup>2</sup>b](#), [3<sup>2</sup>:4](#), [\(A<sub>4</sub>×3\):2](#)[2].
51.  $S_3 \times 3a$  [140], Nor  $S_3^2a$ , Der 3b, Orb 6, 1. **Contains 14 groups, 8 classes.**  
Above [3<sup>2</sup>](#), [6c](#)[3], [S<sub>3</sub>c](#). Below [S<sub>3</sub><sup>2</sup>a](#).
52.  $S_3 \times 3b$  [140], Nor  $S_3^2b$ , Der 3a, Orb 3<sup>2</sup>, 1. **Contains 14 groups, 8 classes.**  
Above [3<sup>2</sup>](#), [6b](#)[3], [S<sub>3</sub>a](#). Below [S<sub>3</sub><sup>2</sup>b](#), [A<sub>4</sub>×S<sub>3</sub>](#), [S<sub>4</sub>×3](#).

## 15. THE 3 CLASSES OF SUBGROUPS OF ORDER 20 AND INDEX 252.

53.  $5:4a$  [126], Nor  $5:4 \times 2$ , Der 5, Orb 5, 2. **Contains 14 groups, 6 classes.**  
Above [D<sub>10</sub>b](#), [4b](#)[5]. Below [5:4×2](#), [S<sub>5</sub>b](#).
54.  $D_{20}$  [126], Nor  $5:4 \times 2$ , Der 5, Orb 5, 2. **Contains 22 groups, 10 classes.**  
Above [10](#), [D<sub>10</sub>b](#), [D<sub>10</sub>a](#), [2<sup>2</sup>e](#)[5]. Below [5:4×2](#), [A<sub>5</sub>×2](#).
55.  $5:4b$  [126], Nor  $5:4 \times 2$ , Der 5, Orb 5, 1<sup>2</sup>. **Contains 14 groups, 6 classes.**  
Above [D<sub>10</sub>b](#), [4a](#)[5]. Below [5:4×2](#), [S<sub>5</sub>a](#), [S<sub>5</sub>c](#)[2].

## 16. THE CLASS OF SUBGROUPS OF ORDER 21 AND INDEX 240.

56.  $7:3$  [120], Nor 7:6, Der 7, Orb 7. **Contains 10 groups, 4 classes.**  
Above [7](#), [3b](#)[7]. Below [7:6](#), [L<sub>3</sub>\(2\)](#)[2].

## 17. THE 14 CLASSES OF SUBGROUPS OF ORDER 24 AND INDEX 210.

57.  $S_4a$  [35], Nor  $S_4 \times S_3$ , Der  $A_4a$ , Orb 4, 1<sup>3</sup>. **Contains 30 groups, 11 classes. PY.**  
Above [A<sub>4</sub>a](#), [D<sub>8</sub>a](#)[3], [S<sub>3</sub>a](#)[4]. Below [S<sub>4</sub>×2a](#)[3], [S<sub>4</sub>×3](#), [S<sub>5</sub>a](#)[3].
58.  $S_3 \times 2^2a$  [35], Nor  $S_4 \times S_3$ , Der 3a, Orb 4, 3. **Contains 54 groups, 16 classes.**  
Above [D<sub>12</sub>c](#)[3], [D<sub>12</sub>b](#)[3], [2<sup>2</sup>×3a](#), [2<sup>3</sup>a](#)[3]. Below [S<sub>3</sub>×D<sub>8</sub>](#)[3], [A<sub>4</sub>×S<sub>3</sub>](#).
59.  $S_3 \times 2^2b$  [105], Nor  $S_3 \times D_8$ , Der 3a, Orb 3, 2<sup>2</sup>. **Contains 54 groups, 20 classes. NPY.**  
Above [D<sub>12</sub>e](#)[2], [D<sub>12</sub>d](#)[2], [D<sub>12</sub>c](#), [D<sub>12</sub>a](#), [2<sup>2</sup>×3b](#), [2<sup>3</sup>b](#)[3]. Below [S<sub>3</sub>×D<sub>8</sub>](#), [S<sub>5</sub>×2](#)[2].

60.  $D_8 \times 3$  [105], Nor  $S_3 \times D_8$ , Der  $2b$ , Orb 4, 3. Contains 20 groups, 14 classes.  
Above  $12$ ,  $2^2 \times 3b$ ,  $2^2 \times 3a$ ,  $D_8a$ . Below  $S_3 \times D_8$ ,  $S_4 \times 3$ .
61.  $D_{24}$  [105], Nor  $S_3 \times D_8$ , Der  $6a$ , Orb 4, 3. Contains 34 groups, 15 classes.  
Above  $12$ ,  $D_{12b}$ ,  $D_{12a}$ ,  $D_8b[3]$ . Below  $S_3 \times D_8$ .
62.  $A_4 \times 2a$  [105], Nor  $S_4 \times 2a$ , Der  $2^2a$ , Orb 4, 2, 1. Contains 26 groups, 12 classes.  
Above  $A_4a$ ,  $2^3a$ ,  $6b[4]$ . Below  $S_4 \times 2a$ ,  $A_4 \times S_3$ ,  $A_5 \times 2$ .
63.  $S_4b$  [105], Nor  $S_4 \times 2b$ , Der  $A_4c$ , Orb 6, 1. Contains 30 groups, 11 classes.  
Above  $A_4c$ ,  $D_8b[3]$ ,  $S_3c[4]$ . Below  $S_4 \times 2b$ ,  $S_5c[2]$ .
64.  $A_4 \times 2b$  [105], Nor  $S_4 \times 2b$ , Der  $2^2b$ , Orb 6, 1. Contains 26 groups, 12 classes.  
Above  $A_4c$ ,  $2^3b$ ,  $6c[4]$ . Below  $S_4 \times 2b$ .
65.  $2.D_{12a}$  [105], Nor  $S_3 \times D_8$ , Der  $6a$ , Orb 4, 3. Contains 30 groups, 16 classes.  
Above  $2.S_3$ ,  $D_{12b}$ ,  $2^2 \times 3b$ ,  $D_8c[3]$ . Below  $S_3 \times D_8$ .
66.  $S_3 \times 4$  [105], Nor  $S_3 \times D_8$ , Der  $3a$ , Orb 4, 3. Contains 26 groups, 16 classes.  
Above  $12$ ,  $D_{12c}$ ,  $2.S_3$ ,  $4 \times 2[3]$ . Below  $S_3 \times D_8$ .
67.  $2.D_{12b}$  [105], Nor  $S_3 \times D_8$ , Der  $6a$ , Orb 4, 3. Contains 30 groups, 13 classes.  
Above  $2.S_3$ ,  $D_{12a}$ ,  $2^2 \times 3a$ ,  $D_8d[3]$ . Below  $S_3 \times D_8$ ,  $(A_4 \times 3):2$ .
68.  $S_4c$  [105], Nor  $S_4 \times 2a$ , Der  $A_4a$ , Orb 4, 2, 1. Contains 30 groups, 10 classes.  
Above  $A_4a$ ,  $D_8d[3]$ ,  $S_3d[4]$ . Below  $S_4 \times 2a$ ,  $(A_4 \times 3):2$ ,  $S_5b$ ,  $A_6$ .
69.  $S_4d$  [105], Nor  $S_4 \times 2b$ , Der  $A_4c$ , Orb 6, 1. Contains 30 groups, 10 classes.  
Above  $A_4c$ ,  $D_8d[3]$ ,  $S_3e[4]$ . Below  $S_4 \times 2b$ ,  $L_3(2)[2]$ ,  $A_6$ .
70.  $S_4e$  [210], Selfnor, Der  $A_4b$ , Orb 4, 3. Contains 30 groups, 10 classes. N.  
Above  $A_4b$ ,  $D_8d[3]$ ,  $S_3e[4]$ . Below  $(A_4 \times 3):2$ ,  $L_3(2)$ .

## 18. THE 4 CLASSES OF SUBGROUPS OF ORDER 36 AND INDEX 140.

71.  $A_4 \times 3$  [35], Nor  $S_4 \times S_3$ , Der  $2^2a$ , Orb 4, 3. Contains 30 groups, 11 classes. D.  
Above  $A_4b[2]$ ,  $2^2 \times 3a$ ,  $A_4a$ ,  $3^2[4]$ . Below  $A_4 \times S_3$ ,  $S_4 \times 3$ ,  $(A_4 \times 3):2$ .
72.  $S_3^2a$  [70], Nor  $S_3 \wr 2$ , Der  $3^2$ , Orb 6, 1. Contains 60 groups, 15 classes. N.  
Above  $S_3 \times 3a[2]$ ,  $3^2:2$ ,  $D_{12f}[6]$ . Below  $S_3 \wr 2$ .
73.  $S_3^2b$  [70], Nor  $S_3 \wr 2$ , Der  $3^2$ , Orb  $3^2$ , 1. Contains 60 groups, 15 classes. NPY.  
Above  $S_3 \times 3b[2]$ ,  $3^2:2$ ,  $D_{12d}[6]$ . Below  $S_3 \wr 2$ ,  $S_4 \times S_3[2]$ .
74.  $3^2:4$  [70], Nor  $S_3 \wr 2$ , Der  $3^2$ , Orb 6, 1. Contains 38 groups, 10 classes.  
Above  $3^2:2$ ,  $4b[9]$ . Below  $S_3 \wr 2$ ,  $A_6$ .

## 19. THE CLASS OF SUBGROUPS OF ORDER 40 AND INDEX 126.

75.  $5:4 \times 2$  [126], Selfnor, Der  $5$ , Orb 5, 2. Contains 40 groups, 16 classes. N.  
Above  $5:4b$ ,  $D_{20}$ ,  $5:4a$ ,  $4 \times 2[5]$ . Below  $S_5 \times 2$ .

## 20. THE CLASS OF SUBGROUPS OF ORDER 42 AND INDEX 120.

76.  $7:6$  [120], Selfnor, Der  $7$ , Orb 7. Contains 26 groups, 8 classes. N.  
Above  $7:3$ ,  $D_{14}$ ,  $6c[7]$ . Below  $S_7$ .

## 21. THE 3 CLASSES OF SUBGROUPS OF ORDER 48 AND INDEX 105.

77.  $S_4 \times 2a$  [105], Selfnor, Der  $A_4a$ , Orb 4, 2, 1. Contains 98 groups, 29 classes. NPY.  
Above  $S_4c$ ,  $A_4 \times 2a$ ,  $S_4a$ ,  $D_8 \times 2[3]$ ,  $D_{12d}[4]$ . Below  $S_4 \times S_3$ ,  $S_5 \times 2$ ,  $S_6$ .
78.  $S_3 \times D_8$  [105], Selfnor, Der  $6a$ , Orb 4, 3. Contains 120 groups, 42 classes. N.  
Above  $2.D_{12b}$ ,  $S_3 \times 4$ ,  $2.D_{12a}$ ,  $D_{24}$ ,  $D_8 \times 3$ ,  $S_3 \times 2^2b$ ,  $S_3 \times 2^2a$ ,  $D_8 \times 2[3]$ . Below  $S_4 \times S_3$ .
79.  $S_4 \times 2b$  [105], Selfnor, Der  $A_4c$ , Orb 6, 1. Contains 98 groups, 29 classes. N.  
Above  $S_4d$ ,  $A_4 \times 2b$ ,  $S_4b$ ,  $D_8 \times 2[3]$ ,  $D_{12f}[4]$ . Below  $S_6$ .

## 22. THE 2 CLASSES OF SUBGROUPS OF ORDER 60 AND INDEX 84.

80.  $A_5a$  [21], Nor  $S_5 \times 2$ , Perfect, Orb 5,  $1^2$ . Contains 59 groups, 9 classes. D.  
Above  $A_4a[5]$ ,  $D_{10}b[6]$ ,  $S_3d[10]$ . Below  $S_5a$ ,  $A_5 \times 2$ ,  $S_5b$ ,  $A_6[2]$ .
81.  $A_5b$  [42], Nor  $S_5c$ , Perfect, Orb 6, 1. Contains 59 groups, 9 classes. D.  
Above  $A_4c[5]$ ,  $D_{10}b[6]$ ,  $S_3e[10]$ . Below  $S_5c$ ,  $A_6$ .

## 23. THE 4 CLASSES OF SUBGROUPS OF ORDER 72 AND INDEX 70.

82.  $A_4 \times S_3$  [35], Nor  $S_4 \times S_3$ , Der  $2^2 \times 3a$ , Orb 4, 3. Contains 94 groups, 25 classes.  
Above  $A_4 \times 3$ ,  $A_4 \times 2a[3]$ ,  $S_3 \times 2^2a$ ,  $S_3 \times 3b[4]$ . Below  $S_4 \times S_3$ .
83.  $S_4 \times 3$  [35], Nor  $S_4 \times S_3$ , Der  $A_4a$ , Orb 4, 3. Contains 70 groups, 23 classes.  
Above  $A_4 \times 3$ ,  $D_8 \times 3[3]$ ,  $S_4a$ ,  $S_3 \times 3b[4]$ . Below  $S_4 \times S_3$ .
84.  $(A_4 \times 3):2$  [35], Nor  $S_4 \times S_3$ , Der  $A_4 \times 3$ , Orb 4, 3. Contains 140 groups, 23 classes. N.  
Above  $A_4 \times 3$ ,  $S_4e[6]$ ,  $S_4c[3]$ ,  $2.D_{12}b[3]$ ,  $3^2:2[4]$ . Below  $S_4 \times S_3$ ,  $A_7$ .
85.  $S_3:2$  [70], Selfnor, Der  $3^2:2$ , Orb 6, 1. Contains 112 groups, 26 classes. N.  
Above  $3^2:4$ ,  $S_3^2b$ ,  $S_3^2a$ ,  $D_8c[9]$ . Below  $S_6$ .

## 24. THE 4 CLASSES OF SUBGROUPS OF ORDER 120 AND INDEX 42.

86.  $S_5a$  [21], Nor  $S_5 \times 2$ , Der  $A_5a$ , Orb 5,  $1^2$ . Contains 156 groups, 19 classes. PY.  
Above  $A_5a$ ,  $S_4a[5]$ ,  $5:4b[6]$ ,  $D_{12}d[10]$ . Below  $S_5 \times 2$ ,  $S_6[2]$ .
87.  $A_5 \times 2$  [21], Nor  $S_5 \times 2$ , Der  $A_5a$ , Orb 5, 2. Contains 164 groups, 22 classes.  
Above  $A_5a$ ,  $A_4 \times 2a[5]$ ,  $D_{20}[6]$ ,  $D_{12}e[10]$ . Below  $S_5 \times 2$ .
88.  $S_5b$  [21], Nor  $S_5 \times 2$ , Der  $A_5a$ , Orb 5, 2. Contains 156 groups, 17 classes.  
Above  $A_5a$ ,  $S_4c[5]$ ,  $5:4a[6]$ ,  $D_{12}a[10]$ . Below  $S_5 \times 2$ ,  $A_7$ .
89.  $S_5c$  [42], Selfnor, Der  $A_5b$ , Orb 6, 1. Contains 156 groups, 19 classes. N.  
Above  $A_5b$ ,  $S_4b[5]$ ,  $5:4b[6]$ ,  $D_{12}f[10]$ . Below  $S_6$ .

## 25. THE CLASS OF SUBGROUPS OF ORDER 144 AND INDEX 35.

90.  $S_4 \times S_3$  [35], Selfnor, Der  $A_4 \times 3$ , Orb 4, 3. Contains 372 groups, 60 classes. NPY.  
Above  $(A_4 \times 3):2$ ,  $S_4 \times 3$ ,  $A_4 \times S_3$ ,  $S_3 \times D_8[3]$ ,  $S_4 \times 2a[3]$ ,  $S_3^2b[4]$ . Below  $S_7$ .

## 26. THE CLASS OF SUBGROUPS OF ORDER 168 AND INDEX 30.

91.  $L_3(2)$  [30], Selfnor, Perfect, Orb 7. Contains 179 groups, 15 classes. DN.  
Above  $S_4e[7]$ ,  $S_4d[7]$ ,  $7:3[8]$ . Below  $A_7$ .

## 27. THE CLASS OF SUBGROUPS OF ORDER 240 AND INDEX 21.

92.  $S_5 \times 2$  [21], Selfnor, Der  $A_5a$ , Orb 5, 2. Contains 535 groups, 49 classes. NPY.  
Above  $S_5b$ ,  $A_5 \times 2$ ,  $S_5a$ ,  $S_4 \times 2a[5]$ ,  $5:4 \times 2[6]$ ,  $S_3 \times 2^2b[10]$ . Below  $S_7$ .

## 28. THE CLASS OF SUBGROUPS OF ORDER 360 AND INDEX 14.

93.  $A_6$  [7], Nor  $S_6$ , Perfect, Orb 6, 1. Contains 501 groups, 22 classes. D.  
Above  $A_5b[6]$ ,  $A_5a[6]$ ,  $3^2:4[10]$ ,  $S_4d[15]$ ,  $S_4c[15]$ . Below  $S_6$ ,  $A_7$ .

## 29. THE CLASS OF SUBGROUPS OF ORDER 720 AND INDEX 7.

94.  $S_6$  [7], Selfnor, Der  $A_6$ , Orb 6, 1. Contains 1455 groups, 56 classes. NPY.  
Above  $A_6$ ,  $S_5c[6]$ ,  $S_5a[6]$ ,  $S_3:2[10]$ ,  $S_4 \times 2b[15]$ ,  $S_4 \times 2a[15]$ . Below  $S_7$ .

## 30. THE CLASS OF SUBGROUPS OF ORDER 2520 AND INDEX 2.

95.  $A_7$  [1], Normal, Perfect, Orb 7. Contains 3786 groups, 37 classes. D.  
Above  $A_6[7]$ ,  $L_3(2)[30]$ ,  $S_5b[21]$ ,  $(A_4 \times 3):2[35]$ . Below  $S_7$ .

## 31. THE CLASS OF SUBGROUPS OF ORDER 5040 AND INDEX 1.

96.  $S_7$  [1], Selfnor, Der  $A_7$ , Orb 7. Contains 11300 groups, 96 classes. NPY.  
Above  $A_7$ ,  $S_6[7]$ ,  $S_5 \times 2[21]$ ,  $S_4 \times S_3[35]$ ,  $7:6[120]$ .