

# THE SUBGROUPS OF $W(H_3)$ .

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### 1. THE CLASS OF SUBGROUPS OF ORDER 1 AND INDEX 120.

**1. 1 [1], Normal, Orb  $1^{30}$ .** Contains 1 group in 1 class. DY.  
Below [2a](#), [2b](#)[15], [2c](#)[15], [3](#)[10], [5](#)[6].

### 2. THE 3 CLASSES OF SUBGROUPS OF ORDER 2 AND INDEX 60.

**2. 2a [1], Normal, Orb  $2^{15}$ .** Contains 2 groups in 2 classes.  
Above [1](#). Below [2<sup>2</sup>b](#)[15], [6](#)[10], [10](#)[6].

**3. 2b [15], Nor  $2^3$ , Orb  $2^{14}, 1^2$ .** Contains 2 groups in 2 classes.  
Above [1](#). Below [2<sup>2</sup>a](#), [2<sup>2</sup>b](#), [2<sup>2</sup>c](#), [S<sub>3</sub>b](#)[2], [D<sub>10a</sub>](#)[2].

**4. 2c [15], Nor  $2^3$ , Orb  $2^{13}, 1^4$ .** Contains 2 groups in 2 classes. Y.  
Above [1](#). Below [2<sup>2</sup>b](#), [2<sup>2</sup>c](#)[2], [S<sub>3a</sub>](#)[2], [D<sub>10b</sub>](#)[2].

### 3. THE CLASS OF SUBGROUPS OF ORDER 3 AND INDEX 40.

**5. 3 [10], Nor  $D_{12}$ , Orb  $3^{10}$ .** Contains 2 groups in 2 classes. D.  
Above [1](#). Below [S<sub>3a</sub>](#), [6](#), [S<sub>3b</sub>](#), [A<sub>4</sub>](#)[2].

### 4. THE 3 CLASSES OF SUBGROUPS OF ORDER 4 AND INDEX 30.

**6. 2<sup>2</sup>a [5], Nor  $A_4 \times 2$ , Orb  $4^6, 2^3$ .** Contains 5 groups in 3 classes. D.  
Above [2b](#)[3]. Below [2<sup>3</sup>](#), [A<sub>4</sub>](#).

**7. 2<sup>2</sup>b [15], Nor  $2^3$ , Orb  $4^6, 2^3$ .** Contains 5 groups in 5 classes.  
Above [2c](#), [2b](#), [2a](#). Below [2<sup>3</sup>](#), [D<sub>12</sub>](#)[2], [D<sub>20</sub>](#)[2].

**8. 2<sup>2</sup>c [15], Nor  $2^3$ , Orb  $4^6, 2^2, 1^2$ .** Contains 5 groups in 4 classes. Y.  
Above [2c](#)[2], [2b](#). Below [2<sup>3</sup>](#).

## 5. THE CLASS OF SUBGROUPS OF ORDER 5 AND INDEX 24.

9.  $5$  [6], Nor  $D_{20}$ , Orb  $5^6$ . **Contains** 2 groups in 2 classes. D.  
Above [1](#). Below [D<sub>10a</sub>](#), [D<sub>10b</sub>](#), [10](#).

## 6. THE 3 CLASSES OF SUBGROUPS OF ORDER 6 AND INDEX 20.

10.  $S_3a$  [10], Nor  $D_{12}$ , Der 3, Orb  $6^3, 3^4$ . **Contains** 6 groups in 4 classes. Y.  
Above [3](#), [2c](#)[3]. Below [D<sub>12</sub>](#).

11.  $6$  [10], Nor  $D_{12}$ , Orb  $6^5$ . **Contains** 4 groups in 4 classes.  
Above [3](#), [2a](#). Below [D<sub>12</sub>](#),  $A_4 \times 2$ [2].

12.  $S_3b$  [10], Nor  $D_{12}$ , Der 3, Orb  $6^4, 3^2$ . **Contains** 6 groups in 4 classes.  
Above [3](#), [2b](#)[3]. Below [D<sub>12</sub>](#),  $A_5$ .

## 7. THE CLASS OF SUBGROUPS OF ORDER 8 AND INDEX 15.

13.  $2^3$  [5], Nor  $A_4 \times 2$ , Orb  $8^3, 2^3$ . **Contains** 16 groups in 8 classes. N.  
Above [2<sup>2</sup>c](#)[3], [2<sup>2</sup>b](#)[3], [2<sup>2</sup>a](#). Below  $A_4 \times 2$ .

## 8. THE 3 CLASSES OF SUBGROUPS OF ORDER 10 AND INDEX 12.

14.  $D_{10a}$  [6], Nor  $D_{20}$ , Der 5, Orb  $10^2, 5^2$ . **Contains** 8 groups in 4 classes.  
Above [5](#), [2b](#)[5]. Below [D<sub>20</sub>](#),  $A_5$ .

15.  $D_{10b}$  [6], Nor  $D_{20}$ , Der 5, Orb  $10, 5^4$ . **Contains** 8 groups in 4 classes. Y.  
Above [5](#), [2c](#)[5]. Below [D<sub>20</sub>](#).

16.  $10$  [6], Nor  $D_{20}$ , Orb  $10^3$ . **Contains** 4 groups in 4 classes.  
Above [5](#), [2a](#). Below [D<sub>20</sub>](#).

## 9. THE 2 CLASSES OF SUBGROUPS OF ORDER 12 AND INDEX 10.

17.  $A_4$  [5], Nor  $A_4 \times 2$ , Der  $2^2a$ , Orb  $12^2, 6$ . **Contains** 10 groups in 5 classes.  
Above [2<sup>2</sup>a](#), [3](#)[4]. Below  $A_4 \times 2$ ,  $A_5$ .

18.  $D_{12}$  [10], Selfnor, Der 3, Orb  $12, 6^3$ . **Contains** 16 groups in 10 classes. N.  
Above [S<sub>3b</sub>](#), [6](#), [S<sub>3a</sub>](#), [2<sup>2</sup>b](#)[3]. Below  $W(H_3)$ .

## 10. THE CLASS OF SUBGROUPS OF ORDER 20 AND INDEX 6.

19.  $D_{20}$  [6], Selfnor, Der 5, Orb  $10^3$ . **Contains** 22 groups in 10 classes. N.  
Above [10](#), [D<sub>10b</sub>](#), [D<sub>10a</sub>](#), [2<sup>2</sup>b](#)[5]. Below  $W(H_3)$ .

## 11. THE CLASS OF SUBGROUPS OF ORDER 24 AND INDEX 5.

20.  $A_4 \times 2$  [5], Selfnor, Der  $2^2a$ , Orb 24, 6. **Contains** 26 groups in 12 classes. N.  
Above [A<sub>4</sub>](#), [2<sup>3</sup>](#), [6](#)[4]. Below  $W(H_3)$ .

## 12. THE CLASS OF SUBGROUPS OF ORDER 60 AND INDEX 2.

21.  $A_5$  [1], Normal, Perfect, Orb 30. **Contains** 59 groups in 9 classes. D.  
Above  $A_4$ [5],  $D_{10a}$ [6],  $S_3b$ [10]. Below  $W(H_3)$ .

## 13. THE CLASS OF SUBGROUPS OF ORDER 120 AND INDEX 1.

22.  $W(H_3)$  [1], Selfnor, Der  $A_5$ , Orb 30. **Contains** 164 groups in 22 classes. NY.  
Above [A<sub>5</sub>](#),  $A_4 \times 2$ [5],  $D_{20}$ [6],  $D_{12}$ [10].