Week 1: Introduction, Sets, Inequalities.

MA161/MA1161: Semester 1 Calculus.

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Welcome to MA161.

MA161 is **Mathematical Studies**: a mathematics module for students in 1st year Science.
(**MA1161** is **Mathematical Studies** for 1st year Project and Construction Management.)

MA161 has 4 parts:

1. **Semester 1 Calculus.** Mon and Tue.
2. **Semester 1 Algebra.** Wed and Thu.
3. **Semester 2 Calculus.**
4. **Semester 2 Algebra/Probability/Statistics.**

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Are You In the Right Class?

Remember: MA161 is for students who are not qualified for MA180 ("Honours Mathematics").

This course requires the same level of involvement and commitment as MA180 - same number and style of homework assignments, tests, etc.

The main difference is that it is focused on Mathematics needed to study other science subjects, whereas MA180 can prepare you to take more specialist courses in Mathematics.

Both MA161 and MA180 are also available through Irish.

ARE YOU SURE YOU ARE IN THE RIGHT MODULE?
It is not too late to change!

What, Where and When.

Lectures: Mon 1:00–1:50pm in the O'Flaherty (Calculus)
Tue 10:00–10:50am in the Kirwan (Calculus).
Wed 10:00–10:50am in the Kirwan (Algebra).
Thu 10:00–10:50am in the Kirwan (Algebra).

Tutorials: **to be arranged . . .**

Web Site: There are on-line resources for MA161 Semester 1 Calculus at
`http://schmidt.nuigalway.ie/ma161-1`
and at this course's blackboard page.

There you'll find various pieces of information, including

- these slides (but not the notes that are hand-written in class),
- homework assignments,
- announcements,
- etc.

Course Assessment.

Your progress in and commitment to this course will be assessed as follows:

- **Continuous Assessment:** There will be exercises given every week in lectures. These are for you to work on, on your own. Versions of these will form the basis for the Calculus part of the on-line homework assignments. Results from these count as the Continuous Assessment (CA) component of MA161.
  CA contibutes 40% to the overall mark.
- **Exams:** There will be a 2 hour written exam for Algebra and Calculus at the end of each semester.
  Each exam contributes 30% to the overall mark.

What is Calculus?

From Wikipedia:

Calculus is the mathematical study of **change**.

It has two major branches, **differential calculus** (concerning **rates of change** and **slopes of curves**), and **integral calculus** (concerning **accumulation** of quantities and the **areas** under and between curves); these two branches are related to each other by the **fundamental theorem of calculus**. Both branches make use of the fundamental notions of **convergence** of certain **infinite sequences** and **series** to a well-defined **limit**. Today, calculus has widespread uses in **science**, **engineering** and **economics** and can solve many problems that algebra alone cannot. Calculus is an important part of **modern mathematics education**. A course in calculus is a gateway to other, more advanced courses in mathematics.

Topics, Textbook.

The key topics in MA161 Semester 1 Calculus are:

1. Sets and Functions,
2. Limits and Derivatives,
3. Differentiation Techniques,
4. Applications of Differentiation.

The recommended textbook is:

James Stewart
Calculus: Early Transcendentals 7th Edition.

There are copies in the library. Also, it can be bought in the College Bookshop or online.

A Short Review of Sets

A **set**, roughly speaking, is a **collection of objects**. If the set is made up of, for example, the numbers 2, 4 and 8, we write it as

$$\{2, 4, 8\}.$$

If we want to give the set a **name**, for example, call it S, we write

$$S = \{2, 4, 8\}.$$

When talking about the set S, we say things like

- 4 is an **element** (or member) of S,
- S contains 4,

and we write

$$4 \in S.$$

Subsets.

We call a set T a **subset** of the set S if every element of T is also an element of S (that is, if S contains every member of T).

Example

Let $S = \{2, 4, 8\}$ and $T = \{2, 8\}$. Then T is a subset set of S. We write $T \subseteq S$.

Exercises.

Let $S = \{-1, 2, 7, 13\}$. Which of the following are subsets of S?

- $T_1 = \{-1, 7, 13\}$,
- $T_2 = \{1, 2, 13\}$,
- $T_3 = \{13, 7, 2, -1\}$,
- $T_4 = \{-1, 2, 7, 13, 15\}$,
- $T_5 = \{2\}$,
- $T_6 = 7$.

The Natural Numbers.

The most fundamental (or natural) set of numbers is the **Natural Numbers**:

$$\mathbb{N} = \{1, 2, 3, \ldots\}.$$

These are the ones we use for **counting**.

But they are not sufficient for solving even simple linear equations such as:

Find all x such that $x + 3 = 1$.

The solution to this, $x = -2$, is a **negative** number (which is not a natural number) . . .

The Integers.

The next bigger set of numbers is the set of **Integers**

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

Note that every natural number is also an integer. Mathematically, we write this as $\mathbb{N} \subseteq \mathbb{Z}$. In English we read this as “the Natural numbers are a subset of the set of Integers”.

But even the Integers are not a big enough set to contain the solutions to even very simple equations. For example:

Find all x such that $3x - 2 = 0$.

The solution to this, $x = 2/3$, is a **fraction** (which is not an integer).

The Rationals.

The next set of numbers we'll look at is called the **Rationals**, denoted by the symbol $\mathbb{Q}$, which is made up of all the numbers are can be expressed as fractions. More precisely:

$$\mathbb{Q} = \{\tfrac{p}{q} \mid p \in \mathbb{Z},\ q \in \mathbb{N}\}.$$

This means that a number is **rational** if can be written as the quotient of an integer and a natural number ($\neq 0$).

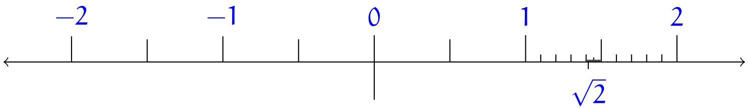
Examples

- $3\frac{3}{7} = \frac{24}{7}$ is rational.
- $-2.14 = \frac{-107}{50}$ is rational.
- $\pi = 3.14159265358979\ldots$ is **not** rational.

Note that $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$.

The Real Numbers.

Finally, there are the **real numbers**, denoted by $\mathbb{R}$. We use the real numbers for **measuring** things (weight, length, speed, etc.). The (real) **number line** is the line whose points are the real numbers:



To be very precise, we could say that real numbers are limits of sequences of rational numbers such as

$$\sqrt{2} = 1.41421356\ldots$$
$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

The set $\mathbb{R}$ of real numbers is set of numbers we are most concerned about in this course.

Note that $\mathbb{Q} \subseteq \mathbb{R}$, ...

Inequalities.

We will frequently need to solve problems of the following type:

Find all real numbers x such that

1. $x + 1 > 0$,
2. $1 - x \geqslant 0$,
3. $|x + 1| \leqslant 1$,
4. $e^x \leqslant 1$,
5. $\ldots$

So we'll spend some time now working out how to solve these, and how to express the answers.

Intervals.

Interval Notation.

Certain subsets of the real numbers are called **intervals**. We write:

- $[a, b]$ for the set of all x such that $a \leqslant x \leqslant b$; this set is called a **closed interval**, i.e., we **include** both end-points
- (a, b) for the set of all x such that $a < x < b$; this set is called a **open interval**, i.e., we **exclude** both end-points
- $[a, \infty)$ for the set of all numbers greater or equal to a, i.e., $x \geqslant a$.

Examples

- All x **strictly** between 3 and 5:
  $x \in (3, 5)$, or $3 < x < 5$.
- All x less than -1:
  $x \in (-\infty, -1)$, or $x < -1$.
- All x greater than or equal to 4:
  $x \in [4, \infty)$, or $x \geqslant 4$.

Solving Inequalities.

We'll solve some inequalities, starting with simple **linear** examples.

1. Find all numbers x such that $5x + 1 > 0$.
   (Soln: $x \in (-\frac{1}{5}, \infty)$.)
2. Find all numbers x such that $-\frac{1}{2}x + 4 \leqslant 0$.
   (Soln: $x \in [8, \infty)$.)
3. Find all numbers x such that $3x + 1 \geqslant 4x + 2$.
   (Soln: $x \in (-\infty, -1]$.)

More Inequalities.

Next, we'll solve some more complicated **nonlinear** inequalities. Here it is helpful to remember that, for two numbers a and b,

- if $ab = 0$ then at least one of a and b equals 0;
- if $ab < 0$ then one of a and b is positive and one is negative;
- if $ab > 0$ then either both numbers are positive, or both are negative.

1. Find all real numbers x such that $x^2 \geqslant 0$.
2. Find all real numbers x such that $x^3 \leqslant 0$.
3. Find all real numbers x such that $x^3 + x^2 < 0$.
   (Soln: $\{x : x < -1\} = (-\infty, -1)$.)
4. Find all real numbers x such that $x^2 - 1 \geqslant 0$.
5. Find all real numbers x such that $x^2 + 1 \leqslant 0$.
6. (From the MA161 Semester 1 Exam, 2013/2014.) Solve the following inequality: $x^2 > x + 6$.

Example.

Example (MA161 Semester 1 Exam, 2013/2014.)

Solve the following inequality: $x^2 > x + 6$.

Solution:

1. Write the inequality as $x^2 - x - 6 > 0$.
2. Solve the quadratic equation: $x^2 - x - 6 = 0$ iff $x = 3$ or $x = -2$.
3. Factorize the quadratic: $x^2 - x - 6 = (x - 3)(x + 2)$.
4. if $ab > 0$ then EITHER $a > 0$ and $b > 0$, OR $a < 0$ and $b < 0$: EITHER $x - 3 > 0$ and $x + 2 > 0$, OR $x - 3 < 0$ and $x + 2 < 0$.
5. EITHER $x > 3$ and $x > -2$, OR $x < 3$ and $x < -2$.
6. Either $x > 3$ or $x < -2$.
7. Solution: $x \in (-\infty, -2) \cup (3, \infty)$.

Symbols

The following frequently used symbols are worth remembering:

$\{\ \}$	encloses sets
$\in$	is an element of
$\subseteq$	is a subset of
$\mathbb{N}$	the natural numbers
$\mathbb{Z}$	the integers
$\mathbb{Q}$	the rationals
$\mathbb{R}$	the reals
$<$	(strictly) less than
$\leqslant$	less than or equal
$[\ ]$	closed interval
$(\ )$	open interval

Exercises.

1. Go to the library. Find where they keep the calculus books. Choose any three. Find the section where they introduce the concept of a **function**. Write down the **definition** of a function that they provide, their **explanation** of what it means, and one **example**. Rank the books in order of how useful you think they are.
2. The study of what we call "Calculus" is said to have been started by **Isaac Newton** and **Gottfried von Leibniz**. Find out **when and where they lived**, and what their major mathematical discoveries were.
3. Solve the following inequalities:

$$\text{(i)}\quad 4 - x^2 \leqslant 0, \qquad \text{(ii)}\quad x^2 \geqslant x + 2, \qquad \text{(iii)}\quad \tfrac{1}{2}x^2 < x^3.$$

4. What sets are usually represented by the symbols $\mathbb{R}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{C}$? For each one, determine which of the others it is a subset of.