1a) $Q(x) = x^3$, $b = 3$

$P(x) = 3^x$, $a = 3$

$R(x) = x^{-1}$, $c = -1$

1b) i) $18 - 2x^2 = 2(9 - x^2)$

$= 2(3 - x)(3 + x) \leq 0$

$\Leftrightarrow x \leq -3$ or $x \geq 3$

ii) $1 + 2|x - 3| > 7$

$2|x - 3| > 6$

$|x - 3| > 3$

$$|x - 3| = \begin{cases} -(x-3) & \text{if } x - 3 < 0 \text{, i.e. } x < 3 \\ x - 3 & \text{if } x - 3 \geq 0 \text{, i.e. } x \geq 3 \end{cases}$$

So: if $x < 3$, we need $-(x-3) > 3$

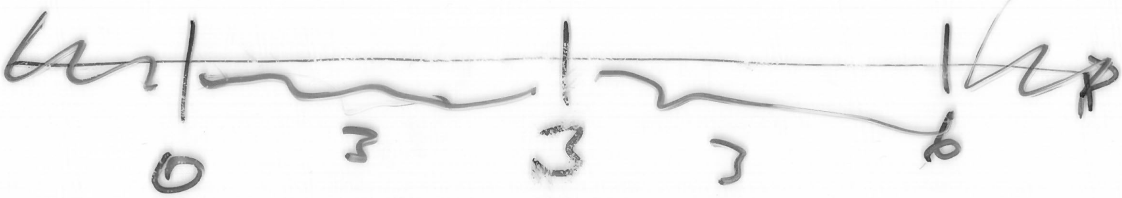
$x - 3 < -3$

$x < 0$.

if $x \geq 3$, we need $x - 3 > 3$

$x > 6$

Ans: $x < 0$ ~~or~~ $x > 6$



1 c) i) Note $f(0) = 100 \cdot \left(\frac{1}{2}\right)^0 = 100$ ✓

Then: $f(t+4) = 100 \left(\frac{1}{2}\right)^{(t+4)/4}$

$= 100 \left(\frac{1}{2}\right)^{(t/4 + 1)}$

$= 100 \left(\frac{1}{2}\right)^{t/4} \cdot \frac{1}{2} = \frac{1}{2} f(t)$ ✓

ii) Find t such that $f(t) \leq 1$.

$$100 \left(\frac{1}{2}\right)^{t/4} \leq 1$$

$$\left(\frac{1}{2}\right)^{t/4} \leq \frac{1}{100}$$

$$2^{t/4} \geq 100$$

$$\log_2\left(2^{t/4}\right) \geq \log_2 100 \approx 6.64$$

$$t/4 \geq 6.64$$

$$t \geq 26.6$$

Ans. 27 days.

1d) even: $f(-x) = f(x)$

odd: $f(-x) = -f(x)$

i) $f(-x) = -1 - (-x)^2 - (-x)^4$

$= -1 - x^2 - x^4 = f(x)$

f is <u>even</u>

ii) $f(-x) = e^{-x} - e^{-(-x)}$

$= -(e^x - e^{-x}) = -f(x)$

f is <u>odd</u>

2 a) i) $\lim_{x \to -8} \dfrac{\overbrace{x^2 + 11x + 24}^{f(x)}}{x+8}$

$\underset{\left[\frac{0}{0}\right]}{=} \lim_{x \to -8} \dfrac{\overbrace{2x + 11}^{f'(x)}}{\underbrace{1}_{g'(x)}} = \underline{\underline{-5}}$

by l'Hôpital's Rule.

ii) $\lim_{x \to \infty} \dfrac{x^4 - 4x^2 + 4}{x^5 + 4x^3}$ divide by highest power of x in denominator

$= \lim_{x \to \infty} \dfrac{x^4/x^5 - 4x^2/x^5 + 4/x^5}{x^5/x^5 + 4x^3/x^5}$

$= \lim_{x \to \infty} \dfrac{1/x - 4/x^3 + 4/x^5}{1 + 4/x^2}$

$= \dfrac{0 - 0 + 0}{1 + 0} = \underline{\underline{0}}$

by the Limit Laws.

2(d) i)

$f(x)$ is continuous at $x=0$

if $f(x)$ is defined at $x=0$

and if $\lim_{x \to 0} f(x)$ exists

and if $\lim_{x \to 0} f(x) = f(0)$.

$f(x) = \frac{x^3}{|x|}$ is not defined at $x=0$.

Therefore $f(x)$ is not continuous at $x=0$.

2b) ii) Check one-sided limits at $x=-1$ and $x=1$.

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1}(4x^2-2x+1) = 7$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1}(6-x) = 7 \quad ✓$$

f <u>is</u> continuous at $x=-1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(6-x) = 5$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(1+(1+x)^2) = 5 \quad ✓$$

f <u>is</u> continuous at $x=1$

2c) $$\frac{f(x+h)-f(x)}{h} = \frac{2\sqrt{x+h}-2\sqrt{x}}{h}$$

$$= \frac{2}{h}\,\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{2}{h}\,\frac{x+h-x}{\sqrt{x+h}+\sqrt{x}} = \frac{2}{\sqrt{x+h}+\sqrt{x}}$$

$$\Rightarrow \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \frac{2}{\sqrt{x}+\sqrt{x}} = \frac{1}{\sqrt{x}}$$

3a) i) $f(x) = u(v(x))$ when

$u(v) = \ln v$, $v(x) = x^2+1$.

Chain Rule:

$$f'(x) = \frac{du}{dv}\cdot\frac{dv}{dx} = \frac{1}{v}\cdot(2x) = \frac{2x}{x^2+1}.$$

ii) $$f'(x) = \frac{-(x+1)\sin(x^3+2)\cdot 3x^2 - \cos(x^3+2)}{(x+1)^2}$$

using the Quotient Rule and the Chain Rule.

3b) $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx} 25$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$\leadsto$ slope $-3/4$ at $(x,y) = (3,4)$.

$\leadsto$ tangent : $y - 4 = -\frac{3}{4}(x-3)$

$$\underline{y = -\frac{3}{4}x + \frac{25}{4}}$$

c) $f(t) = 10 + 5t - \ln(1+40t)$

$f'(t) = 5 - \frac{40}{1+40t} = 0 \iff \frac{40}{1+40t} = 5$

$\iff t = 7/40$ (critical point)

$f(0) = 10 - \ln(1) = 10$

$f(1) = 15 - \ln(41) \approx 11.286$

$f(7/40) \approx 8.7955$.

$\underline{\text{Ans}}$: minimum 8.7955 at $t = 7/40$

maximum 11.286 at $t = 1$.