

$$1a) \quad Q(x) = x^3, \quad b = 3$$

$$P(x) = 3x^1, \quad a = 3$$

$$R(x) = x^{-1}, \quad c = -1$$

$$1b) \quad i) \quad 18 - 2x^2 = 2(9 - x^2)$$

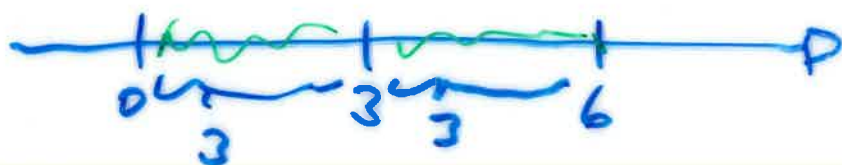
$$= 2(3 - x)(3 + x) \leq 0$$

$$\Leftrightarrow x \leq -3 \quad \text{or} \quad x \geq 3$$

$$ii) \quad 1 + 2|x - 3| > 7$$

$$2|x - 3| > 6$$

$$|x - 3| > 3$$



Aus: $x < 0$ or $x > 6$

$$1c) \quad i) \quad \text{Note: } f(0) = 100 \cdot \left(\frac{1}{2}\right)^0 = 100 \quad \checkmark$$

$$\text{And: } f(t+4) = 100 \cdot \left(\frac{1}{2}\right)^{(t+4)/4}$$

$$= 100 \cdot \left(\frac{1}{2}\right)^{t/4 + 1}$$

$$= 100 \cdot \left(\frac{1}{2}\right)^{t/4} \cdot \frac{1}{2} = \frac{1}{2} f(t) \quad \checkmark$$

ii) Find t such that $f(t) \leq 1$:

$$100 \cdot \left(\frac{1}{2}\right)^{t/4} \leq 1$$

$$\left(\frac{1}{2}\right)^{t/4} \leq \frac{1}{100}$$

$$2^{t/4} \geq 100$$

$$\log_2(2^{t/4}) \geq \log_2 100 \approx 6.64$$

$$t/4 \geq 6.64$$

$$t \geq 26.6$$

Ans: 27 days

1d) even: $f(-x) = f(x)$
odd: $f(-x) = -f(x)$

i) $f(-x) = -1 - (-x)^2 - (-x)^4$
 $= -1 - x^2 - x^4 = f(x)$
f is even

ii) $f(-x) = e^{-x} - e^{-(-x)}$
 $= -(e^x - e^{-x}) = -f(x)$
f is odd

1e) i) $\lim_{x \rightarrow -8} \frac{x^2 + 11x + 24}{x + 8}$

$\frac{0}{0}$
 $\lim_{x \rightarrow -8} \frac{2x + 11}{1} = \underline{-5}$ (l'Hôpital)

ii) $\lim_{x \rightarrow \infty} \frac{x^4 - 4x^2 + 4}{x^5 + 4x^3}$

$$= \lim_{x \rightarrow \infty} \frac{1/x - 4/x^3 + 4/x^5}{1 + 4/x^2}$$

$$= \frac{0 - 0 + 0}{1 + 0} = \underline{0} \text{ (Limit Law)}$$

2a) i) $f(x)$ is continuous at $x=0$
if $f(x)$ is defined at $x=0$.
and if $\lim_{x \rightarrow 0} f(x)$ exists and
if $\lim_{x \rightarrow 0} f(x) = f(0)$.

$f(x) = \frac{x^3}{|x|}$ is not defined at $x=0$.

Therefore $f(x)$ is not continuous at $x=0$.

ii) Check one-sided limits at
 $x = -1$ and $x = 1$:

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1} (4x^2 - 2x + 1) = 7$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1} (6 - x) = 7 \quad \checkmark$$

f is continuous at $x = -1$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} (6 - x) = 5$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} (1 + (1+x)^2) = 5 \quad \checkmark$$

f is continuous at $x = 1$

$$\begin{aligned}
 2b) \quad & \frac{f(x+h) - f(x)}{h} = \frac{2\sqrt{x+h} - 2\sqrt{x}}{h} \\
 & = \frac{2}{h} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{\sqrt{x+h} + \sqrt{x}} \\
 & = \frac{2}{h} \frac{x+h-x}{\sqrt{x+h} + \sqrt{x}} = \frac{2}{\sqrt{x+h} + \sqrt{x}}
 \end{aligned}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \frac{2}{\sqrt{x} + \sqrt{x}} = \frac{1}{\sqrt{x}}$$

\triangleleft i) $f(x) = u(v(x))$ where
 $u(v) = \ln v, \quad v(x) = x^2 + 1.$

Chain Rule:

$$f'(x) = \frac{du}{dv} \cdot \frac{dv}{dx} = \frac{1}{v} \cdot 2x = \frac{2x}{x^2 + 1}$$

ii) $f(x) = \frac{u(x)}{v(x)}$, where

$$u(x) = \cos(x^3 + 2), \quad v(x) = x + 1$$

$$u'(x) = -3x^2 \sin(x^3 + 2) \quad (\text{Chain Rule})$$

Quotient Rule: $f'(x) = \frac{u'v - uv'}{v^2}$

$$f'(x) = \frac{-3x^2 \sin(x^3 + 2) \cdot (x + 1) - \cos(x^3 + 2)}{(x + 1)^2}$$

$$2d) \frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 25$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

at $(x, y) = (3, 4)$ the curve has slope $-\frac{3}{4}$.

tangent: $y - 4 = -\frac{3}{4}(x - 3)$, i.e.,

$$y = -\frac{3}{4}x + \frac{25}{4}.$$

$$2e) \text{ if } f(t) = 10 + 5t - \ln(1 + 40t)$$

$$f'(t) = 5 - \frac{40}{1+40t} = 0 \Leftrightarrow \frac{40}{1+40t} = 5$$

$$\Leftrightarrow t = \frac{7}{40} \quad (\text{critical point})$$

$$f(0) = 10 - \ln(1) = 10$$

$$f(1) = 15 - \ln(41) = 11.286$$

$$f\left(\frac{7}{40}\right) = 8.7955$$

Ans: minimum 8.7955 at $t = \frac{7}{40}$

maximum 11.286 at $t = 1$.

ii)

