

Problem Sheet 1

1. Solve the following sets of simultaneous linear equations:

$$\begin{aligned} \text{(i)} \quad 3x - y &= 9 \\ 5x - 2y &= 13 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 2x + y &= 4 \\ 7x + 5y &= 11 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad 5x - y &= -16 \\ 8x - 3y &= -27 \end{aligned}$$

2. Let

$$A = \begin{pmatrix} 5 & 0 \\ 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 2 \\ 2 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}.$$

- (i) Compute $A + B$, $A - C$, $3A$, $-2B$, $0C$, $2A - 5B$.
- (ii) Verify that $A + (B + C) = (A + B) + C$, $(A \pm B)^t = A^t \pm B^t$, $(2A - 5B)^t = 2A^t - 5B^t$.
- (iii) Solve for X the matrix equations $A + X = B$, $2B - X = 3C$, $4A + 2X = -C$.

3. Compute the determinants

$$\text{(i)} \quad \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix}$$

$$\text{(ii)} \quad \begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix}$$

$$\text{(iii)} \quad \begin{vmatrix} 2 & -6 \\ -3 & 9 \end{vmatrix}$$

4. Let

$$A = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -6 & 4 \\ -3 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & -4 \\ 3 & -6 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad v = \begin{pmatrix} -3 \\ 5 \end{pmatrix}.$$

- (i) Compute AB , BA , AC , CA , BC , CB , A^2 , B^2 , Au , Bv .
- (ii) Verify that $(AB)C = A(BC)$, $(BA)C = B(AC)$, $A(3u - 4v) = 3Au - 4Av$.
- (iii) Verify that $(AB)^t = B^t A^t$, $(CA)^t = A^t C^t$, $(BC)^t = C^t B^t$.
- (iv) Verify that $(AB)^* = B^* A^*$, $(CA)^* = A^* C^*$, $(BC)^* = C^* B^*$.
- (v) Verify that $|AB| = |A| \cdot |B| = |BA|$, $|BC| = |B| \cdot |C| = |CB|$, $|A^2| = |A|^2$. Find $|A^4|$.

5. Let

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 2 & -7 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 \\ -2 & 0 \end{pmatrix}.$$

- (i) Compute A^{-1} , B^{-1} , C^{-1} , $(AB)^{-1}$, $(AC)^{-1}$, $(BC)^{-1}$.
- (ii) Verify that $(AB)^{-1} = B^{-1}A^{-1}$, $(AC)^{-1} = C^{-1}A^{-1}$, $(BC)^{-1} = C^{-1}B^{-1}$.
- (iii) Find the image of the line $L: 3x - 2y - 5 = 0$ under the transformations defined by A and B . Find also a line whose image under A is L .

6. Suppose that $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$. Find $|A|$. Does A^{-1} exist?

Can you find a matrix B so that $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$?

7. Consider the matrices $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$.

- (i) Find, if possible, the inverses A^{-1} and B^{-1} .
- (ii) Determine A^2 , and show that $(A^2)^{-1} = (A^{-1})^2$.
- (iii) Find the image of the line $x + 2y = 2$ under the transformation defined by A . Find also the line whose image under the transformation defined by A is $x + y = 1$.